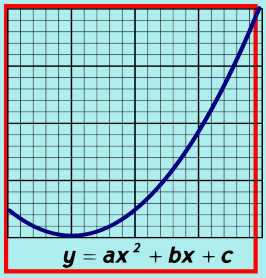
Math 25

Fall 2017

Lecture 15



Graphing Polynomial Function in Factored form:

$f(x) = (x-2)(x+3) = x^2 + x - 6$

Y-Int $\rightarrow x = 0$ $\qquad f(0) = (0-2)(0+3) = -6$

Y-Int $(0,-6)$

x-Int $\rightarrow f(x) = 0 \rightarrow (x-2)(x+3) = 0$

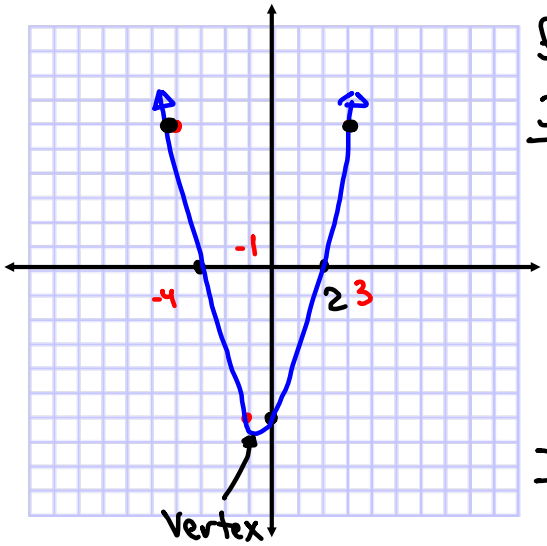
$(2,0)$ & $(-3,0)$ $\qquad x-2=0 \qquad x+3=0$

$x=2 \qquad x=-3$

$f(-4)$

$$\begin{array}{r|rrr} -4 & 1 & 1 & -6 \\ & & -4 & 12 \\ \hline & 1 & -3 & 6 \end{array}$$

$f(-4) = 6$



$f(3)$

$$\begin{array}{r|rrr} 3 & 1 & 1 & -6 \\ & & 3 & 12 \\ \hline & 1 & 4 & 6 \end{array}$$

$f(3) = 6$

$f(-1)$

$$\begin{array}{r|rrr} -1 & 1 & 1 & -6 \\ & & -1 & 0 \\ \hline & 1 & 0 & -6 \end{array}$$

$f(-1) = -6$

$$f(x) = (x-2)\underbrace{(x+4)(x-4)} = (x-2)(x^2-16)$$
$$= x^3 - 2x^2 - 16x + 32$$

Y-Int $\rightarrow x = 0 \rightarrow f(0) = (0-2)(0+4)(0-4)$

$\boxed{f(0) = 32}$ Y-Int $(0, 32)$

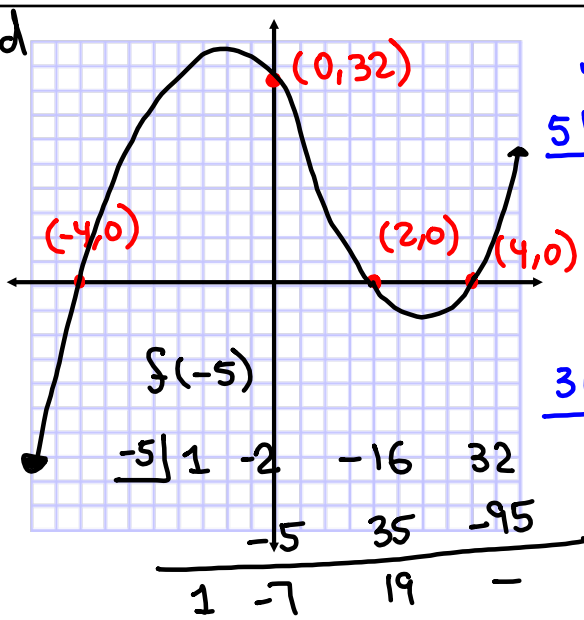
x-Int $\rightarrow f(x) = 0 \rightarrow (x-2)(x+4)(x-4) = 0$

$x - 2 = 0$ $\quad x + 4 = 0$ $\quad x - 4 = 0$

$x = 2$ $\quad x = -4$ $\quad x = 4$

x-Ints $\rightarrow (2,0), (-4,0), (4,0)$

Not Scaled



$f(5) = +$

$$\begin{array}{r|rrrr} 5 & 1 & -2 & -16 & 32 \\ & & 5 & 15 & -5 \\ \hline & 1 & 3 & -1 & 27 \end{array}$$

$f(3) = -$

$$\begin{array}{r|rrrr} 3 & 1 & -2 & -16 & 32 \\ & & 3 & 3 & -39 \\ \hline & 1 & 1 & -13 & - \end{array}$$

$f(-5)$

$$\begin{array}{r|rrrr} -5 & 1 & -2 & -16 & 32 \\ & & -5 & 35 & -95 \\ \hline & 1 & -7 & 19 & - \end{array}$$

$f(-5) = -$

$f(x) = (x-2)^2(x+2)^2$

y-Int $\rightarrow x=0 \rightarrow f(0) = (0-2)^2(0+2)^2$

$= (-2)^2 \cdot (2)^2 = 4 \cdot 4 = 16$

$\boxed{\text{y-Int } (0,16)}$

x-Ints $\longrightarrow y=0 \rightarrow f(x)=0 \rightarrow (x-2)^2(x+2)^2 = 0$

x-Int $(2,0)$ repeated twice

$(-2,0)$ repeated twice

$x-2=0$ $\quad$ $x+2=0$

$\boxed{x=2}$ $\quad$ $\boxed{x=-2}$

Repeated ans.

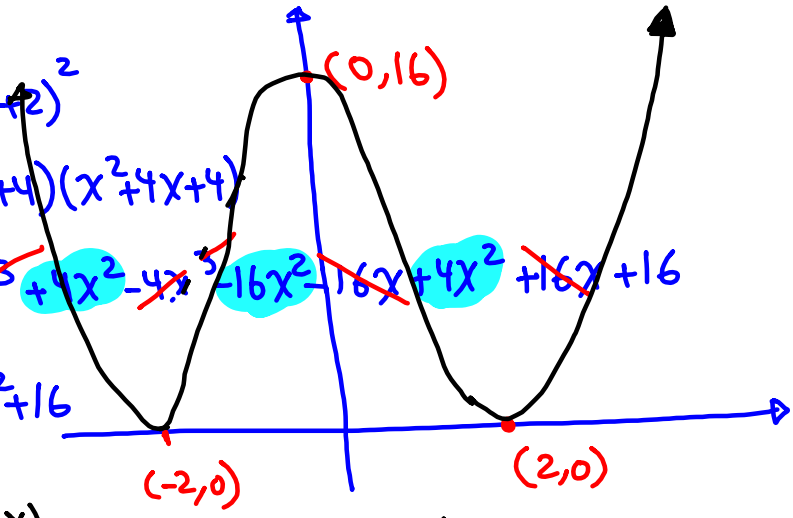
Not Scaled

$f(x) = (x-2)^2(x+2)^2$

$= (x^2-4x+4)(x^2+4x+4)$

$= x^4 + \cancel{4x^3} + 4x^2 - \cancel{4x^3} - 16x^2 - \cancel{16x} + 4x^2 + \cancel{16x} + 16$

$f(x) = x^4 - 8x^2 + 16$



$f(3)$ → $f(x)$ even → symmetric Y-axis

3	1	0	-8	0	16
		3	24	48	+
	1	3	16	+	+

Graph $f(x) = x^5 - 6x^4 + 12x^3 - 12x^2 + 11x - 6$

It will help if this was factored.

List of possible rational zeros is

$$\frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1} \Rightarrow \pm 1, \pm 2, \pm 3, \pm 6$$

Is 1 a zero? Yes

1	1	-6	12	-12	11	-6
		1	-5	7	-5	6
	1	-5	7	-5	6	0

Try again NO

1	1	-5	7	-5	6
		1	-4	3	-2
	1	-4	3	-2	4

Is 2 a Solution? Yes

$$\begin{array}{r|rrrrr} 2 & 1 & -5 & 7 & -5 & 6 \\ & & 2 & -6 & 2 & -6 \\ \hline & 1 & -3 & 1 & -3 & 0 \end{array}$$

try again

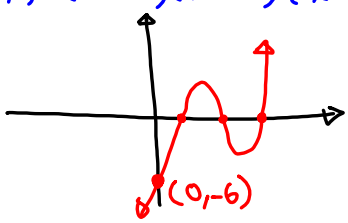
$$\begin{array}{r|rrrr} 2 & 1 & -3 & 1 & -3 \\ & & 2 & -2 & -2 \\ \hline & 1 & -1 & -1 & -5 \end{array}$$

NO

Is 3 a Solution?

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 1 & -3 \\ & & 3 & 0 & 3 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$f(x) = (x-1)(x-2)(x-3)(x^2+1)$



$\rightarrow x^2+1$ is always +

It has no real Soln.

$f(x) = 2x^5 + x^4 + 9x^2 - 32x + 20$

Y-Int $\rightarrow x=0 \rightarrow f(0)=20 \Rightarrow (0,20)$

List all possible rational zeros.

$\underbrace{\pm1, \pm2, \pm4, \pm5, \pm10, \pm20}_{\pm1, \pm2} \Rightarrow \pm1, \pm2, \pm4, \pm5, \pm10, \pm20$

$\pm\frac{1}{2}, \pm\frac{5}{2}$

Show that 1 is a Solution twice.

$$\begin{array}{r|rrrrrr} 1 & 2 & 1 & 0 & 9 & -32 & 20 \\ & & 2 & 3 & 3 & 12 & -20 \\ \hline & 2 & 3 & 3 & 12 & -20 & 0 \end{array} \checkmark$$

$$\begin{array}{r|rrrrr} 1 & 2 & 3 & 3 & 12 & -20 \\ & & 2 & 5 & 8 & 20 \\ \hline & 2 & 5 & 8 & 20 & 0 \end{array} \checkmark$$

Show that $-\frac{5}{2}$ is a solution.

$$\begin{array}{r|rrrr} -\frac{5}{2} & 2 & 5 & 8 & 20 \\ & & -5 & 0 & -20 \\ \hline & 2 & 0 & 8 & 0 \end{array} \checkmark$$

Write $f(x)$ in factored form.

$$f(x) = (x-1)^2\left(x+\frac{5}{2}\right)(2x^2+8)$$

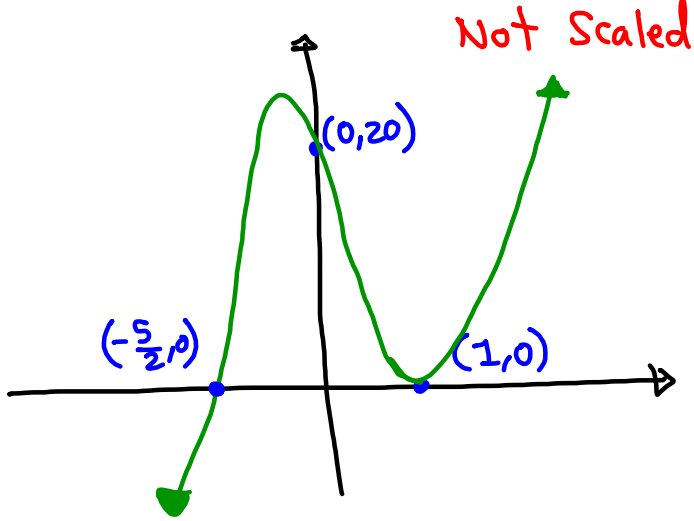
$$f(x) = (x-1)^2\left(x+\frac{5}{2}\right)2(x^2+4)$$

$$f(x) = (x-1)^2(2x+5)(x^2+4)$$

Always +
No real Solns.

Graph

Not Scaled



$f(x) = x^3 - 4x^2 - 2x + 20$

① Find y-Int $(0,20)$

② List all possible rational zeros.

$\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

③ Determine whether or not 2 is a zero.

$$\begin{array}{r|rrrr} 2 & 1 & -4 & -2 & 20 \\ & & 2 & -4 & -12 \\ \hline & 1 & -2 & -6 & 8 \end{array}$$

NO

④ Determine whether or not −2 is a zero

$$\begin{array}{r|rrrr} -2 & 1 & -4 & -2 & 20 \\ & & -2 & 12 & -20 \\ \hline & 1 & -6 & 10 & 0 \end{array}$$

Yes

⑤ use Quadratic formula to find the other two solutions $f(x) = (x+2)(x^2 - 6x + 10)$

$a = 1 \quad b = -6 \quad c = 10$

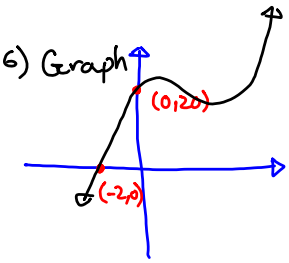
$b^2 - 4ac = (-6)^2 - 4(1)(10)$

$= -4$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow 3 \pm i$

$= \dfrac{6 \pm \sqrt{-4}}{2} = \dfrac{6 \pm 2i}{2}$

⑥ Graph



Graphing Rational Functions

$$f(x) = \frac{\text{Polynomial}}{\text{Polynomial}} \qquad f(x) = \frac{x-2}{x+4}$$

$$f(x) = \frac{x^2 - 9}{2x^2 - 8} \qquad f(x) = \frac{x}{x^2 + 1}$$

Y-Int $\rightarrow x = 0 \rightarrow f(0)$

x-Int $\rightarrow f(x) = 0 \rightarrow$ Numerator $= 0$

Domain $\rightarrow$ All reals except Deno. $= 0$

$f(x) = \dfrac{x-4}{x-2}$

y-Int $\rightarrow x=0$

$f(0) = \dfrac{0-4}{0-2} = \dfrac{-4}{-2} = 2$

Y-Int $(0,2)$

x-Int $\rightarrow f(x)=0 \rightarrow$ Numerator $=0$

$x-4=0 \quad x=4$

x-Int $(4,0)$

Domain: All reals except deno. $=0$

$x-2=0 \quad x=2$

All reals except 2.

Graph $f(x) = \dfrac{x-4}{x-2}$

$f(100) = \dfrac{100-4}{100-2} = \dfrac{96}{98} \approx 1$

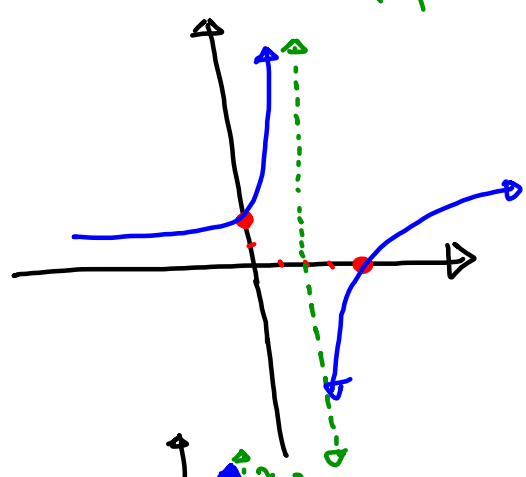
$f(1000) = \dfrac{1000-4}{1000-2} = \dfrac{996}{998} \approx 1$

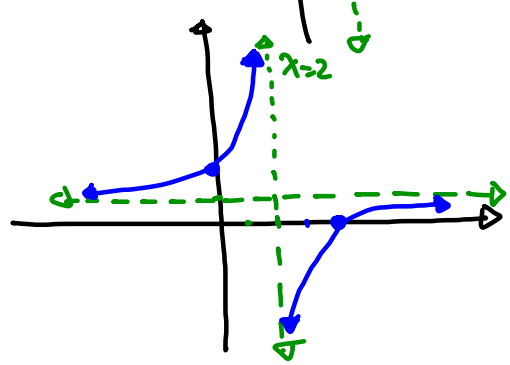
$f(-1000) = \dfrac{-1004}{-1002} \approx 1$

Horizontal Asymptote

$y=1$

Vertical Asymptote





$$f(x) = \frac{2x+6}{x-6}$$

Find domain

All reals except $x-6=0$

6

Find Y-Int $\rightarrow (0,-1)$

$$f(0) = \frac{2(0)+6}{0-6} = -1$$

Find x-Int

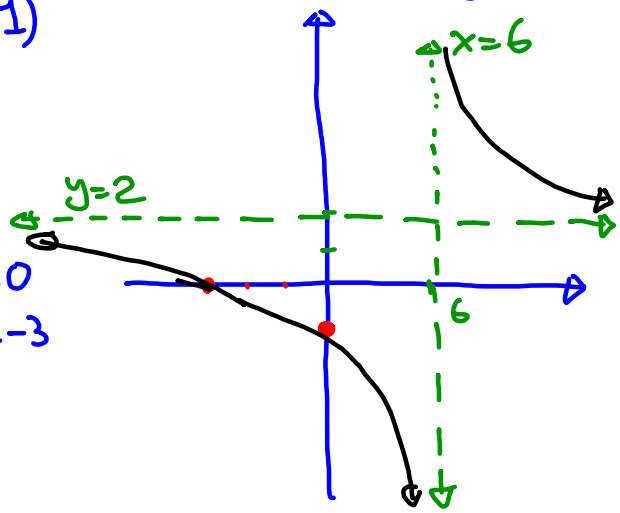
Numerator $= 0$ $\quad 2x+6=0$

$x=-3$

x-Int $(-3,0)$

$$f(100) = \frac{2(100)+6}{100-6} \approx 2$$

$$f(-100) = \frac{-194}{-100-6} \approx 2$$



$$f(x) = \frac{x-4}{x^2-1}$$

③ Domain

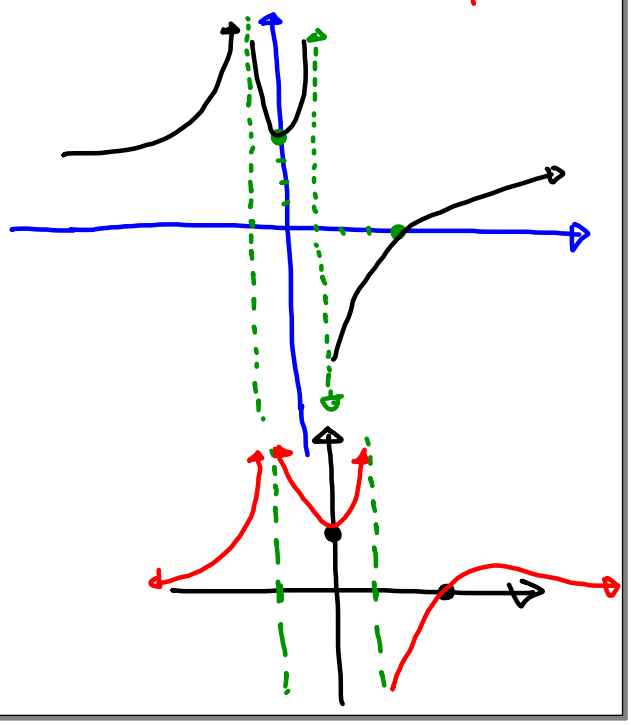
All reals except ± 1

① Y-Int $(0,4)$

② x-Int. $(4,0)$

$$f(100) = \frac{96}{\text{Large \#}} \approx 0$$

$$f(-100) = \frac{-104}{\text{Large \#}} \approx 0$$



$$f(x) = \frac{x}{x^2+4}$$

Not factorable

① Y-Int $(0,0)$

② X-Int $(0,0)$

③ Domain: All Reals

No Vertical Asymptote

④ Find $f(-x)$, what do you conclude?

$$f(-x) = \frac{-x}{(-x)^2+4} = \frac{-x}{x^2+4} = -\frac{x}{x^2+4}$$

$$f(-x) = -f(x)$$

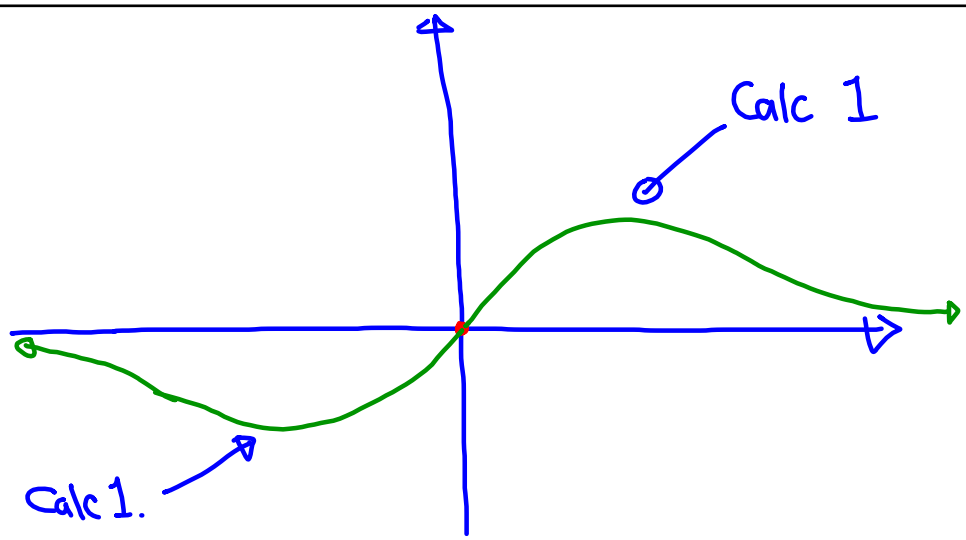
odd function

Symmetric with respect to the origin.

Find $f(100)$ & $f(-100)$

≈ 0

Horizontal Asymptote at $y=0$



$f(x) = \dfrac{8x}{x^2-4}$ $\rightarrow (x+2)(x-2)$

$f(-x) = \dfrac{8(-x)}{(-x)^2-4} = \dfrac{-8x}{x^2-4} = -\dfrac{8x}{x^2-4}$

$= -f(x)$

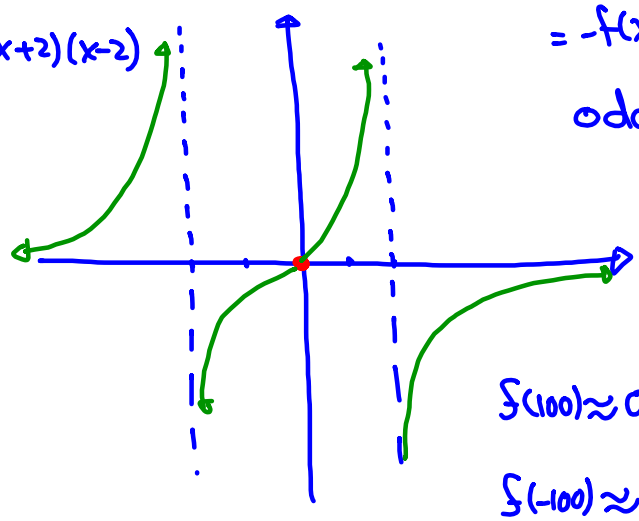
odd

① Y-Int $(0,0)$

③ x-Int $(0,0)$

④ Domain

All Reals except ± 2



$f(100) \approx 0$

$f(-100) \approx 0$

$f(x) = \dfrac{x^2-8x+15}{x^2+2x-3}$

$x^2-8x+15=0$

$(x-3)(x-5)=0$

$x^2+2x-3=0$

$(x+3)(x-1)=0$

$x=-3$ $x=1$

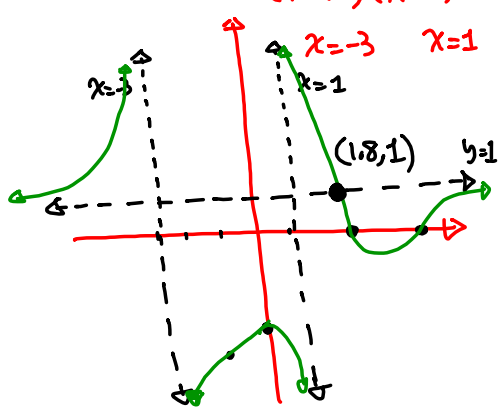
Y-Int $(0,-5)$

x-Int $(3,0), (5,0)$

Domain

All Reals

except $-3, 1$



H.A. $y=1$

$f(x) \to 1$

$\dfrac{x^2-8x+15}{x^2+2x-3} = 1$

$\cancel{x^2}-8x+15 = \cancel{x^2}+2x-3$

$-8x-2x = -3-15$

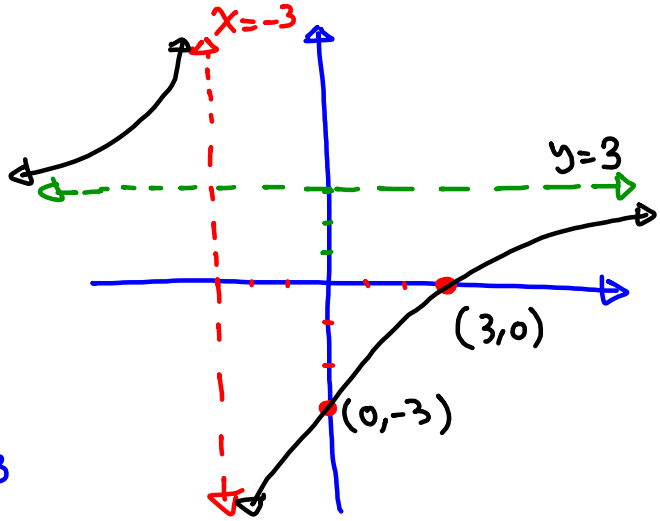
$-10x = -18$ $\boxed{x=1.8}$

$$f(x) = \frac{3x-9}{x+3}$$

y-Int $(0,-3)$

x-Int $(3,0)$

Domain:
All reals except -3



① Calc. OK but no cell phone

② one page of notes, 8.5 by 11, double-sided

③ Review all three exams.

④ Know your graphs

⑤ Review recent notes.

⑥ Make sure you know your logs, your sequences, and mathematical induction.
Binomial Thrm.

⑦ Matrices & Determinant.
↳ inverse of a matrix